Research Methodology

ERROR ANALYSIS ON THE ESTIMATION OF CUMULATIVE INFILTRATION IN SOIL USING GREEN AND AMPT MODEL

(Analisis Kesalahan pada Pendugaan Infiltrasi Kumulatif Menggunakan Model Green and Ampt)

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ABSTRACT

Green and Ampt infiltration model is still useful for the infiltration process because of a clear physical basis of the model and of the existence of the model parameter values for a wide range of soil. The objective of this study was to analyze error on the estimation of cumulative infiltration in soil using Green and Ampt model and to design laboratory experiment in measuring cumulative infiltration. Parameter of the model was determined based on soil physical properties from laboratory experiment. Newton-Raphson method was used to estimate wetting front during calculation using Visual Basic for Application (VBA) in MS Word. The result showed that $\Delta\theta$ contributed the highest error in estimation of cumulative infiltration and was followed by K, H_0, H_f and t respectively. It also showed that the calculated cumulative infiltration is always lower than both measured cumulative infiltration and volumetric soil water content.

Keywords: *Green and Ampt, cumulative infiltration, wetting front, VB Application*

BACKGROUND

Infiltration is the process of water entry into the soil (Hillel, 1980). The rate of this process, relative to the rate of water supply, determines how much water will enter the unsaturated soil zone, and how much, if any, will runoff (Hillel, 1982). Therefore, this soil physical parameter is of paramount importance to the water economy of plant communities, recharge of aquifers, surface runoff, soil erosion, and the fate of pollutants in the environment.

Numerous formulations, some entirely empirically and physically based equations are available to express soil infiltrability as a function of time or the total quantity of water infiltrated into the soil at a point level.

A simple model, which is approximately still usefull for the infiltration process, was suggested by Green and Ampt (Hillel, 1980). The Green and Ampt infiltration model applies combining Darcy's Law and the principle of conservation of mass. It is used because of a clear physical basis of the model and of the existence of Green and Ampt parameter values for a wide range of soil

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(Rawls and Brakensiek, 1983 *in* Yu, 1999). In addition, the major attraction of the Green and Ampt model is the involvement of physically meaningful parameters and the potential of using data on soi1 properties collected during standard soi1 surveys in order to estimate these infiltration parameters. Mein and Larson (1973) *in* Dingman (2002) have been successfully tested the prediction of this model against numerical solution of the Richards Equation.

The objective of this study is to analyze error on the estimation of cumulative infiltration in soil using Green and Ampt model and to design laboratory experiment in measuring cumulative infiltration.

THEORETICAL REVIEW

A. Cumulative Infiltration

With gravity taken into account, the Green and Ampt approach gives (Hillel, 1980):

$$\frac{dI}{dt} = \Delta\theta \frac{dL_f}{dt} = K\frac{H_o - H_f + L_f}{L_f} \quad (1)$$

which integrates to

$$\frac{K}{\Delta\theta} t = L_f - (H_o - H_f)\, L_n\left(1 + \frac{L_f}{(H_o - H_f)}\right) \quad (2)$$

which I is the cumulative infiltration, K is the hydraulic conductivity of the transmission zone, $\Delta\theta = \theta_t - \theta_i$ is the wetness increment between the transmission zone wetness during infiltration and the initial profile wetness, H_0 is the pressure head at the entry surface, H_f is the effective pressure head at the wetting front, L_f is the distance from the surface to the wetting front (the length of the wetted zone) and t is cumulative time elapsed.

Since a uniformly wetted zone is assumed to extend all the way to the wetting front, it follows that

$$I = L_f \Delta\theta \quad (3)$$

B. Error Analysis

The sum of the absolute values of the error terms gives the maximum possible value of the error. However, combining these terms by a root-mean-square and changing to the finite difference form (Jordan *et al.*, 1991) of the equation (3), gives

$$\Delta t = \sqrt{\begin{aligned}&\left(\frac{\partial I}{\partial \Delta\theta}\Delta(\Delta\theta)\right)^2 + \left(\frac{\partial I}{\partial H_o}\Delta H_o\right)^2 \\ &+ \left(\frac{\partial I}{\partial H_f}\Delta H_f\right)^2 + \left(\frac{\partial I}{\partial K}\Delta K\right)^2 \\ &+ \left(\frac{\partial I}{\partial t}\Delta t\right)^2\end{aligned}} \quad (4)$$

which is

$$\frac{\partial I}{\partial \Delta\theta} = \frac{\partial I}{\partial L_f}\frac{\partial L_f}{\partial \Delta\theta} = -\frac{K}{\Delta\theta}\, t\, \frac{(H_o - H_f + L_f)}{L_f} \quad (5)$$

$$\frac{\partial I}{\partial H_o} = \frac{\partial I}{\partial L_f}\frac{\partial L_f}{\partial H_o} = \Delta\theta\left(\frac{H_o - H_f + L_f}{L_f} L_n\left(1 + \frac{L_f}{H_o - H_f}\right) - 1\right) \quad (6)$$

$$\frac{\partial I}{\partial H_f} = \frac{\partial I}{\partial L_f}\frac{\partial L_f}{\partial H_f} = \Delta\theta\left(-\frac{H_o - H_f + L_f}{L_f} L_n\left(1 + \frac{L_f}{H_o - H_f}\right) + 1\right) \quad (7)$$

$$\frac{\partial I}{\partial K} = \frac{\partial I}{\partial L_f}\frac{\partial L_f}{\partial K} = t\frac{(H_o - H_f + L_f)}{L_f} \quad (8)$$

$$\frac{\partial I}{\partial t} = \frac{\partial I}{\partial L_f}\frac{\partial L_f}{\partial t} = K\frac{(H_o - H_f + L_f)}{L_f} \quad (9)$$

EXPERIMENTAL DESIGN

This study was carried out at soil physics and mechanics laboratory, Department of Agricultural Engineering, Bogor Agricultural University.

A Materials

The infiltration test was conducted on silty clay soil type (2 mm-sieved). The soil sample would be manually compacted into soil ring started from lower part of the soil column. Equation (10) below would be used to calculate weight of the soil needed to be compacted into the fixed volume of the soil ring in order to get the expected dry bulk density. The compaction of the soil into soil ring should be done gently enough in order not to destroy soil aggregate.

$$W_s = (W + 1)\, \rho_b\, V \qquad (10)$$

which W_s is soil weight, W is mass wetness, ρ_b is dry bulk density, and V is volume of the container.

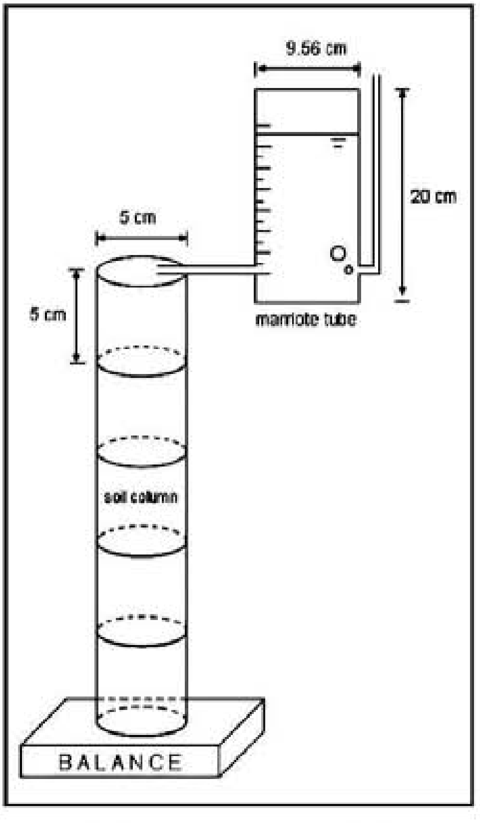

Figure 1. Schematic diagram of infiltration apparatus

used in infiltration experiment is shown in Figure 1. The soil column of the apparatus had 5 cm of diameter and 25 cm of length consisting of 5 pieces of 5-cm soil rings.

B. Methods

Equation (2) consists of 6 variables, which are K, $\Delta\theta$, t, L_f, H_0, and H_f as mentioned above. According to those, the four variables can be derived both from soil properties analysis and infiltration measurement. H_0 is measured by scale during the infiltration measurement, K is derived from hydraulic conductivity measurement, $\Delta\theta$ and H_f are derived from water retention data. The relationship between soil water content and pressure head is derived by using van Genuchten equation (Setiawan, 1992). Another variable, L_f, must be estimated at the certain time t. Since the equation (2) is a nonlinear algebraic equation, a Newton-Raphson numerical solution (Burden and Faires, 1993) was used to estimate L_f.

We applied a marriote tube in order to supply water into the soil surface. The size of the water supplier tube was 9.56 cm of diameter and 20 cm of height, with scale on the wall of tube in order to read water level in the tube. We also use a weighting balance in order to measure the change of soil column weight during the water was infiltrated.

Other soil physical properties required in this study are hydraulic conductivity and water retention data. We use the falling head method of hydraulic conductivity analysis (Klute and Dirksen, 1986). Whereas the water retention data was extracted from water retention curve presented by Saleh (2000).

We developed computer program by using Visual Basic for Application in Microsoft Word to calculate the error of analysis and to simulate the cumulative infiltration.

Table 1. $I_{measured}$ and $\Delta\theta$ values resulted from infiltration experiment

t				V	$I_{measured}$	W_T	ΔW_W	$\Delta\theta$
hour	min	sec	Δsec					
0.00	0.00	0		0.00	0.0	1247.2		0.00
0.05	3.12	187	187	14.35	0.7	1259.2	12.0	0.96
0.24	14.50	870	683	28.70	1.5	1272.7	13.5	2.04
0.47	28.17	1690	820	43.05	2.2	1286.8	14.1	3.18
0.70	41.75	2505	815	57.40	2.9	1301.4	14.6	4.35
0.93	55.83	3350	845	71.74	3.7	1316.1	14.7	5.52
1.17	70.00	4200	850	86.09	4.4	1330.6	14.5	6.69

Remarks:
t is elapsed time, V is accumulative volume of water added (cm^3), Imeasured is cumulative infiltration (cm), WT is accumulative weight of soil sample (gram), ΔWW is weight change of water (gram), and $\Delta\theta$ is change of volumetric water content (cm^3/cm^3).

Table 2. Input data for error analysis in computer simulation

t (hour)	$\Delta\theta$ cm3/cm3	$\Delta\Delta\theta$	H_0 (cm H_2O)	ΔH_0	H_f (cm H_2O)	ΔH_f	k (cm/sec)	Δk
0.052	0.96	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005
0.242	2.04	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005
0.469	3.18	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005
0.696	4.35	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005
0.931	5.52	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005
1.167	6.69	0.005	0.5	0.005	-100	0.005	0.00020	0.0000005

RESULT AND DISCUSSION

Parameter of equation (10) that were resulted from the experiment and obtained from literature are 38.72% of mass wetness, 1 gram/cm^3 of dry bulk density and 98.13 cm^3 of container volume. Having conducted the experiment, the result of infiltration experiment is shown in Table 1.

We can see from Tabel 1 that the longer elapsed time we have, the more water volume is added. Therefore, $I_{measured}$ and $\Delta\theta$ will increase.

By using the parameters of Green and Ampt infiltration model mentioned above, we use the data shown in Table 2 in order to simulate the error analysis of model variables. Having simulated the computer program, we obtained the output data of error analysis shown in Table 3. Based on error analysis, we found the error percentage of each variable.

Based upon completion the error analysis, it showed that $\Delta\theta$ contributed the highest error in cumulative infiltration and was followed by K, H_0, H_f and t respectively. According to Table 1 and

Table 3. Output data from error analysis in computer simulation

t (hour)	Δθ m3/cm3	I (cm)	ΔI	Error Total %	Percentage of individual error				
					Δθ	H_0	H_f	K	t
0.052	0.96	0.294	0.083	28.348	28.347	0.002	0.002	0.136	0.003
0.242	2.04	0.965	0.133	13.824	13.823	0.002	0.002	0.141	0.001
0.469	3.18	1.702	0.153	8.975	8.974	0.002	0.002	0.143	0.000
0.696	4.35	2.44	0.161	6.593	6.591	0.002	0.002	0.143	0.000
0.931	5.52	3.191	0.166	5.212	5.210	0.002	0.002	0.144	0.000
1.167	6.69	3.943	0.17	4.31	4.308	0.002	0.002	0.144	0.000

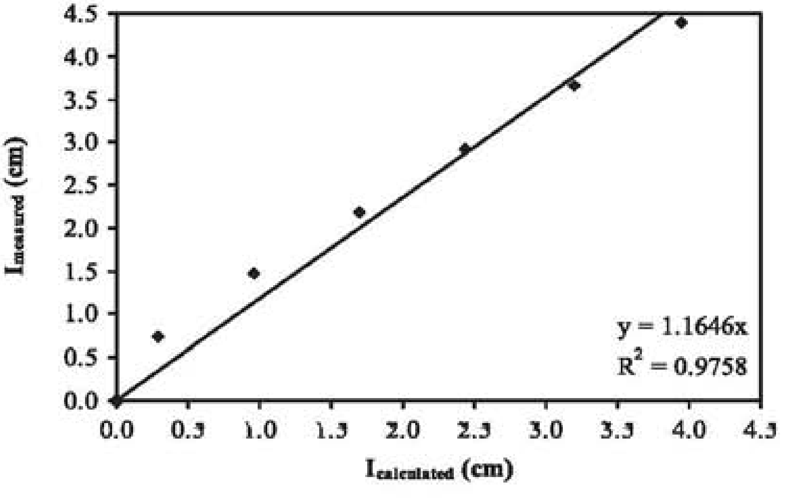

Figure 2. The comparison between calculated and measured cumulative infiltration

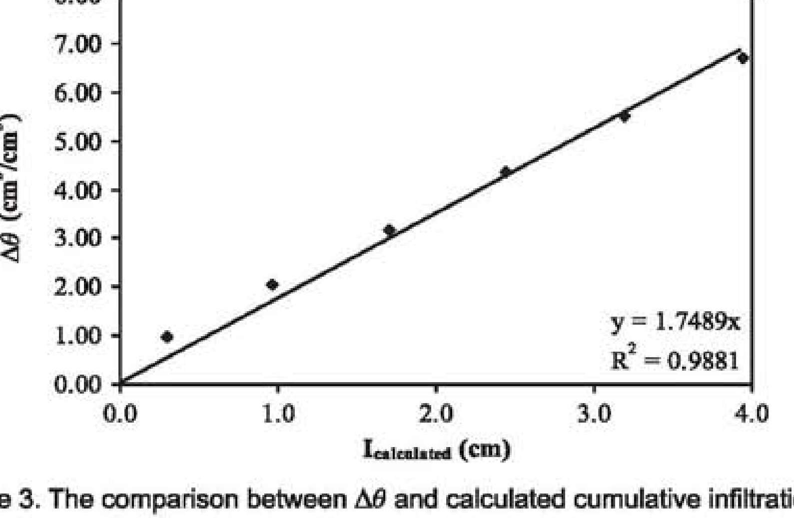

Figure 3. The comparison between Δθ and calculated cumulative infiltration

Table 3, we can compare the calculated and measured cumulative infiltration shown in Figure 2.

Because $I_{measured}$ was obtained by reading the decreasing level of water in the tube manually while volumetric soil water content $\Delta\theta$ was measured by reading scale on the screen of the weighting balance, therefore the latter data seems more accurate. Furthermore, it is better to compare those data with $I_{calculated}$ shown in Figure 3.

According to Figure 2 and 3 above, we can find that the calculated cumulative infiltration is always lower than both measured cumulative infiltration and volumetric soil water content.

CONCLUSION

1. The error analysis of Green and Ampt infiltration model shown that $\Delta\theta$ contributed the highest error in cumulative infiltration and was followed by K, H_0, H_f and t respectively.

2. Error analysis demonstrates the usefulness of error estimation in selecting instrumentation

3. The calculated cumulative infiltration is always lower than both measured cumulative infiltration and volumetric soil water content.

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Appendix

Computer program for error analysis on the estimation of cumulative infiltration in soil using Green and Ampt model