OPTIMAL TRACKING AND REGULATION ACCURACY OF FEEDBACK CONTROL SYSTEMS

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ABSTRACT. This paper deals with intrinsic performance limits achievable by feedback control. We give analytical expressions of the optimal tracking and regulation problems for linear shift-invariant single-input and multiple-output (SIMO) discrete-time systems. For the former, we modify the existing results by means of the delta operator and show that the continuous-time counterpart results can be properly recovered from this point. For the latter, we derive a discrete-time result first and show the convergence property.

Keywords: Performance limits, $\mathcal{H}_2$ optimal control, SIMO discrete-time systems, delta operator.

1. INTRODUCTION

The study on control performance limits achievable by feedback control systems is one of the important research topics in control theory, and it has been the subjects of research for many years. Recently, there has been growing attention devoted to the studies on the optimal tracking and regulation problem. The optimal tracking ability is measured by the minimal tracking error between its output and a reference input to be tracked via a stabilizing compensator, while the latter is measured by the plant input energy.

Existing results on the optimal tracking problem include the continuous-time and discrete-time systems, possibly for SISO, SIMO, and MIMO cases, for instance see [1, 2, 3, 5, 7]. Possible extension is made by considering sampled-data control systems [4]. While, in the optimal regulation problem, existing results available for continuous-time systems [3, 5]. There are at least two issues may arise toward the results. First, the relationship between continuous-time and discrete-time results is not quite clear. Second, we have no discrete-time result on regulation problem. The main reason of this lack might be, in the optimal regulation problem we have to involve a certain function evaluated at infinity which is laid on the $j\omega$-axis, i.e. boundary for s-domain, but not on the unit circle, i.e. boundary for z-domain.

FIGURE 1. The unity feedback control system

Our primary objective in this work is twofold. For the tracking problem, we revisit the SIMO discrete-time results by implementing the delta operator [6] to reveal the unified representation between continuous-time results [2] and discrete-time results [1]. The reason is that, it has been extensively demonstrated that the delta operator is superior to the shift operator for discrete-time systems. For the regulation problem, we derive an analytical expression of the minimal input energy pertaining to the SIMO discrete-time control systems. This serves as the discrete-time counterpart result of [5]. Based on this result we then perform a unified approach by means of the delta operator.

Our results prove that by using a unified approach we can completely recover the continuous-time results from the delta-type results stand point. Additionally, our result on regulation energy problem shows an unusual fact: the unstable poles of the plant contribute effects not by summation but product way.

The rest of this paper is organized as follows. In Section 2 we introduce the feedback control system setup and state some preliminaries. Section 3 is devoted to the tracking problem. We first give two key lemmas, and we reformulate the tracking performance problem by delta operator and then show the unification result. In Section 4, we derive an analytical closed form solution for the energy regulation problem and its unified result. Some concluding statements are in Section 5.

2. Preliminaries

We briefly describe the notation used throughout this paper. We denote the complex plane by $\mathbb{C}$. For any $a \in \mathbb{C}$, its complex conjugate is denoted by $\bar{a}$. For any vector u we shall use u^T, u^H, and $\|u\|$ as its transpose, conjugate transpose, and Euclidean norm, respectively. For any matrix $A \in \mathbb{C}^{m \times n}$, we denote its conjugate transpose by A^H and its column space by $\mathbb{R}[A]$. Several subsets in the complex plane are defined as follows: $\mathbb{C}_- := \{s \in \mathbb{C} : \operatorname{Re} s < 0\}$, $\mathbb{C}_+ := \{s \in \mathbb{C} : \operatorname{Re} s > 0\}$, $\bar{\mathbb{C}}_+ := \{s \in \mathbb{C} : \operatorname{Re} s \geq 0\}$, $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$, $\mathbb{D}^c := \{z \in \mathbb{C} : |z| \geq 1\}$, $\bar{\mathbb{D}}^c := \{z \in \mathbb{C} : |z| > 1\}$.

2.1. **Feedback Control Systems.** The standard setup under consideration in this paper is the discrete-time SIMO feedback system depicted in Fig. 1, where P represents the plant and K the compensator. The signals $r \in \mathbb{R}^m$, $d \in \mathbb{R}$, $u \in \mathbb{R}$, and $y \in \mathbb{R}^m$ are the reference input, the disturbance input, the plant input, and the system output, respectively.

For the plant rational transfer function P, its left and right coprime factorization be given by

$$P = NM^{-1} = \tilde{M}^{-1}\tilde{N}, \tag{1}$$

where $N, M, \tilde{N}, \tilde{M} \in \mathbb{R}\mathcal{H}_\infty$ and they satisfy the double Bezout identity

$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & Y \\ N & X \end{bmatrix} = I \tag{2}$$

for some $X, Y, \tilde{X}, \tilde{Y} \in \mathbb{R}\mathcal{H}_\infty$. Here we define $\mathbb{R}\mathcal{H}_\infty$ be a class of proper stable rational transfer function matrices. Then all the stabilizing compensators K can be characterized by Youla parameterization

$$\begin{aligned} \mathcal{K} \quad := \quad & \{K : K = (Y - MQ)(NQ - X)^{-1} \\ & = (Q\tilde{N} - \tilde{X})^{-1}(\tilde{Y} - Q\tilde{M}); \ Q \in \mathbb{R}\mathcal{H}_\infty\}. \end{aligned} \tag{3}$$

A number $\eta \in \mathbb{C}$ is said to be *zero* of P if $P_i(\eta) = 0$ holds for some $i = 1, \ldots, m$. In addition, if η is lying in $\bar{\mathbb{D}}^c$, then η is said to be *a non-minimum phase zero*. P is said to be minimum phase if it has no non-minimum phase zero; otherwise, it is said to be non-minimum phase. A number $\lambda \in \mathbb{C}$ is said to be a *pole* of P if $P(\lambda)$ is unbounded. If λ is lying in $\bar{\mathbb{D}}^c$, then λ is *an unstable pole* of P. We say P is stable if it has no unstable pole; otherwise, unstable. An equivalent statement for pole λ is that $\tilde{M}(\lambda)w = 0$ for some unitary vector w. And w is called *a pole direction* vector associated with λ. For technical reasons, it is assumed that the plant does not have non-minimum phase zeros and unstable poles at the same location. A transfer function N, not necessarily square, is called an *inner* if N is in $\mathbb{R}\mathcal{H}_\infty$ and $N^T(z^{-1})N(z) = I$ for all $z = e^{j\theta}$. A transfer function M is called *outer* if M is in $\mathbb{R}\mathcal{H}_\infty$ and has a right inverse which is analytic in $\bar{\mathbb{D}}^c$. For an arbitrary $P \in \mathbb{R}\mathcal{H}_\infty$,

$$P(z) = \Theta_i(z)\Theta_o(z), \tag{4}$$

where Θ_i is inner and Θ_o is outer, is defined as an *inner-outer factorization* of P. We call Θ_i the *inner factor* and Θ_o the *outer factor*.

2.2. **Delta Transforms.** The delta operator δ is defined by

$$\delta x(k) = \frac{x(k+1) - x(k)}{T},$$

with $T > 0$ is the sampling period. By taking the $\mathcal{Z}$-transform of above equation we obtain

$$\delta\hat{x}(z) = \frac{z-1}{T}\hat{x}(z).$$

Later, the variable δ is used as the delta operator variable and is analogous to the Laplace variable s for continuous-time systems and the $\mathcal{Z}$-transform variable z for discrete-time systems.

For any signal $f_c(t)$, we denote its Laplace transform by $\hat{f}_c(s)$, and for any sequence $f(k)$ we denote its $\mathcal{Z}$-transform by $\hat{f}(z)$. Recall that the

squared $\mathcal{H}_2$-norms in s-domain and z-domain are defined, respectively, by

$$\|\hat{f}_c(s)\|_2^2 \quad := \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} \|\hat{f}_c(j\omega)\|^2 \, d\omega, \tag{5}$$

$$\|\hat{f}(z)\|_2^2 \quad := \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} \|\hat{f}(e^{j\theta})\|^2 \, d\theta. \tag{6}$$

Then we may define the corresponding delta transform by

$$\hat{f}_T(\delta) = T\hat{f}(z)|_{z=T\delta+1} = T \sum_{k=0}^{\infty} f(k)(T\delta+1)^{-k}. \tag{7}$$

The squared $\mathcal{H}_2$-norm of $\hat{f}_T(\delta)$ is defined as

$$\|\hat{f}_T(\delta)\|_2^2 := \frac{1}{2\pi} \int_{-\pi/T}^{\pi/T} \left\| \hat{f}_T \left(\frac{e^{j\omega T}-1}{T} \right) \right\|^2 \, d\omega. \tag{8}$$

We may easily check that the norm (8) converges to the respecting norm in s-domain (5) as T tends to zero. Also we can get Parseval's identity $\|f_T(k)\|_2^2 = \|\hat{f}_T(\delta)\|_2^2$, where $f_T(k) := f(kT)$ and $\|f_T(k)\|_2^2 \triangleq T \sum_{k=0}^{\infty} |f_T(k)|^2$.

Suppose that $F(z)$ is given and define $G(\delta) = F(T\delta+1)$. Then by setting $\omega = \theta/T$, it is easy to show that

$$\|G(\delta)\|_2^2 = \frac{1}{T} \|F(z)\|_2^2. \tag{9}$$

3. Tracking Performance Limits

In subsequent analysis, we use the following notation. For $T > 0$, we define the following sets: $\mathbb{D}_T = \{\delta \in \mathbb{C} : |T\delta+1| < 1\}$, $\bar{\mathbb{D}}_T = \{\delta \in \mathbb{C} : |T\delta+1| \leq 1\}$, $\partial\mathbb{D}_T = \{\delta \in \mathbb{C} : |T\delta+1| = 1\}$, $\bar{\mathbb{D}}_T^c = \{\delta \in \mathbb{C} : |T\delta+1| > 1\}$. It is obvious that $\partial\mathbb{D}_T$ can be seen as a circle centered at $\delta = -1/T$ with radius $1/T$. Respectively, $\mathbb{D}_T$ and $\bar{\mathbb{D}}_T^c$ can be interpreted as areas inside and outside the circle.

3.1. **Two Lemmas.** We begin this part by introducing two lemmas which play important roles in our subsequent analysis. Consider the class of functions in

$$\mathbb{F}_T := \left\{ h : \lim_{R \to \infty} \max_{\theta \in [-\pi/2, \pi/2]} \frac{|h(R\frac{e^{j\theta}-1}{T})|}{R} = 0 \right\}.$$

Now we are ready to modify Lemmas 1 and 2 of [1] in δ-domain.

Lemma 1. *Let $h(\delta) \in \mathbb{F}_T$ and analytic in $\bar{\mathbb{D}}_T^c$. Denote that $h(\frac{e^{j\theta}-1}{T}) = h_1(\theta) + jh_2(\theta)$, where h_1 and h_2 are real and imaginary parts of h,*

respectively. Suppose that $h(\delta)$ is conjugate symmetric, i.e. $h(\delta) = \overline{h(\bar{\delta})}$. Then

$$\frac{h'(0)}{T} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{h_1(\theta) - h_1(0)}{1 - \cos\theta} \, d\theta. \tag{10}$$

Lemma 2. *Let $h(\delta)$ be a meromorphic function[1] in $\bar{\mathbb{D}}_T^c$ and has no zero or pole on $\partial\mathbb{D}_T$. Suppose that $h(\delta)$ is conjugate symmetric and $\log h(\delta) \in \mathbb{F}_T$. Also, suppose that $\zeta_i \in \bar{\mathbb{D}}_T^c, i = 1, \ldots, N_\zeta$, are zeros and $\rho_i \in \bar{\mathbb{D}}_T^c, i = 1, \ldots, N_\rho$, are poles of $h(\delta)$, all counting multiplicities. Provided that $h(0) \neq 0$, then*

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left| \frac{h(\frac{e^{j\theta}-1}{T})}{h(0)} \right| \frac{d\theta}{1 - \cos\theta} =$$

$$\sum_{i=1}^{N_\zeta} \left[1 + \frac{2}{T\zeta_i} \right] - \sum_{i=1}^{N_\rho} \left[1 + \frac{2}{T\rho_i} \right] + \frac{1}{T} \frac{h'(0)}{h(0)}. \tag{11}$$

3.2. **Tracking Error Problem under Control Input Penalty.**

Let the plant P and the compensator K be given by

$$P(\delta) = [P_1(\delta), P_2(\delta), \ldots, P_n(\delta)]^T,$$
$$K(\delta) = [K_1(\delta), K_2(\delta), \ldots, K_n(\delta)],$$

respectively, with $P_i(\delta)$ and $K_i(\delta)$, $i = 1, \ldots, n$, are scalar transfer functions. Suppose that the plant $P(\delta)$ has an inner-outer factorization $P(\delta) = \Theta_i(\delta)\Theta_o(\delta)$.

As the reference input we consider the step function

$$r(k) = \begin{cases} \nu, & k \geq 0 \\ 0, & k < 0 \end{cases}, \quad \hat{r}_T(\delta) = \frac{T\delta + 1}{\delta}\nu, \tag{12}$$

with $\nu = [\nu_1, \nu_2, \ldots, \nu_n]^T \in \mathbb{R}^n$ is a constant vector, and we assume the system is initially at rest and $d(k) = 0$.

We derive a counterpart result in δ-domain. The performance index to be minimized is

$$J_\delta^* = \inf_{K \in \mathcal{K}} T \sum_{k=0}^{\infty} \left(\|e(k)\|^2 + \|u(k)\|^2 \right), \tag{13}$$

which consists of penalties on the error and the control input. The results to the corresponding problems for s-domain and z-domain are respectively found in [5] and [1].

We state our result for marginally stable plants. Lemmas 1 and 2 are exploited to derive the expression. We can extend the problem to unstable plants case in parallel manner as did in [1]. For the finiteness of J_δ^* we make the following assumptions.

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[1]A meromorphic function on an open subset of the complex plane is a function that is analytic in all except a set of isolated points, which are poles of the function.

Assumption 1. *$P(\delta)$ does not have transmission zeros at $\delta = 0$.*

Assumption 2. *For $r(k)$ defined in (12), $\nu \in \mathbb{R}[P(0)]$.*

Assumption 3. *$P(\delta)$ has a pole at $\delta = 0$.*

Theorem 1. *If $P(\delta)$ is marginally stable and it satisfies Assumptions 1–3, then*

$$J_\delta^* = \sum_{i \in \mathbb{I}} \nu_i^2 \sum_{k=1}^{N_i} \left[\frac{2}{\zeta_{ik}} + T \right] + \frac{T}{2\pi} \sum_{i \in \mathbb{I}} \nu_i^2 \int_0^\pi \frac{\log \left[\frac{\|P(\frac{e^{j\theta}-1}{T})\|^2 + 1}{|P_i(\frac{e^{j\theta}-1}{T})|^2} \right]}{1 - \cos \theta} d\theta \quad (14)$$

where $\mathbb{I}$ is an index set defined by $\mathbb{I} := \{i : P_i(0) \neq 0\}$.

Proof. Note that, by Parseval's identity the optimal cost (13) can be represented as

$$J_\delta^* = \inf_{K \in \mathcal{K}} (\|\hat{e}_T(\delta)\|_2^2 + \|\hat{u}_T(\delta)\|_2^2).$$

Then the processes just follow the proof of corresponding theorem in [1]. □

3.3. **Unified Results.** Note that in the above case we do vary the sampling period T, instead of fix it. Then the tracking measure of corresponding continuous-time system $J_c^* = \inf_{K \in \mathcal{K}_c} \int_0^\infty \|e(t)\|^2 dt$ can be fully recovered by evaluating J_δ^* as T tends to zero. To do this, we need to reformulate the second term of RHS of (14) as follow. For the given sampling period T, we denote the associated sampling frequency by $\omega = \theta/T$. From the Taylor expansion we have $(1 - \cos \theta) \approx \frac{1}{2}\theta^2$ and $\sin \theta \approx \theta$. Hence, we obtain $(e^{j\theta} - 1)/T \approx -\omega^2 T/2 + j\omega$ and $T d\theta/[2(1 - \cos \theta)] \approx d\omega/\omega^2$. Finally,

$$\lim_{T \to 0} J_\delta^* = \sum_{i \in \mathbb{I}} \nu_i^2 \sum_{k=1}^{N_i} \frac{2}{\zeta_{ik}} + \frac{1}{\pi} \sum_{i \in \mathbb{I}} \nu_i^2 \int_0^\infty \log \left[\frac{\|P(j\omega)\|^2 + 1}{|P_i(j\omega)|^2} \right] \frac{d\omega}{\omega^2}$$

shows that we completely recover the minimum tracking error of continuous-time systems J_c^*. This expression is coincident with that in [5].

4. Regulation Performance Limits

4.1. **Energy Regulation Problem.** Here we consider a minimal regulation energy problem for stabilization of unstable plants. The problem of interest is to regulate the input $u(k)$ by designing a stabilizing compensator K. Throughout this section we assume on Fig. 1 that $d(k)$ to be an impulse signal, i.e $\hat{d}(z) = 1$, and $r(k) = 0$. Let factorize $P(z)$ as

$$P(z) = P_s(z)B^{-1}(z) = \Theta_i(z)\Theta_o(z)B^{-1}(z) \quad (15)$$

where P_s is stable part of P, Θ_i and Θ_o are the inner and outer factors of P_s, respectively, and

$$B(z) = \prod_{i=1}^{N_p} \frac{z - p_i}{\bar{p}_i z - 1} \tag{16}$$

be the unstable part of the plant P. We denote by p_i, $i = 1, \ldots, N_p$, the unstable poles of P. It is useful to point out that $B(-1) = 1$ and $B(\infty) = \prod_{i=1}^{N_p} \frac{1}{\bar{p}_i}$.

We formulate and solve the optimal performance index

$$E^* = \inf_{K \in \mathcal{K}} \sum_{k=0}^{\infty} \|u(k)\|^2. \tag{17}$$

The following result gives an explicit expression of E^*.

Theorem 2. *Let the plant $P(z)$ be factorized as in (15). Then,*

$$E^* = E_1^* + E_2^*, \tag{18}$$

where

$$E_1^* = \prod_{i=1}^{N_p} |p_i|^2 - 1$$

$$E_2^* = \sum_{i,j \in \mathbb{N}} \frac{(|s_i|^2 - 1)(|s_j|^2 - 1)}{\bar{b}_i b_j (\bar{s}_i s_j - 1)} \bar{\beta}_i \beta_j$$

with

$$b_i = \prod_{j \in \mathbb{N}, j \neq i} \frac{s_i - s_j}{s_i \bar{s}_j - 1},$$

$$\beta_i = B^{-1}(\infty) - B^{-1}(s_i) \tag{19}$$

and $\mathbb{N}$ is an index set defined by $\mathbb{N} := \{i : \tilde{N}(s_i) = 0\}$.

Proof. We only prove for E_1^*. We may express (17) as

$$E^* = \|K(z)S(z)P(z)\hat{d}(z)\|_2^2 = \|Y\tilde{N} - MQ\tilde{N}\|_2^2.$$

It is possible to factorize $M(z)$ as $M(z) = B(z)M_m(z)$, where $M_m(z)$ is the minimum phase part of $M(z)$ and $B(z)$ is all-pass factor. After some processes, we get

$$E^* = \|B^{-1}(\infty) - B^{-1}\|_2^2 + \inf_{Q \in \mathbb{R}\mathcal{H}_\infty} \|B^{-1}(\infty) - R_1 + M_m Q\tilde{N}\|_2^2, \quad (20)$$

for some $R_1 \in \mathbb{R}\mathcal{H}_\infty$. We denote the first term of RHS of (20) by E_1^*. Since $B(z)$ is inner then

$$E_1^* = \|B^{-1}(\infty)B(z) - 1\|_2^2 = \left\| \prod_{i=1}^{N_p} \frac{\bar{p}_i z - |p_i|^2}{\bar{p}_i z - 1} - 1 \right\|_2^2.$$

Next we define

$$E_1(N) = \left\| \prod_{i=1}^{N} \frac{\bar{p}_i z - |p_i|^2}{\bar{p}_i z - 1} - 1 \right\|_2^2.$$

A further calculation gives $E_1(N) = |p_N|^2 E_1(N-1) + |p_N|^2 - 1$. The proof is complete by performing mathematical induction over $E_1(N)$. Considering the following inner function might be very useful in this process

$$\alpha(z) = \frac{\bar{p}_N z - 1}{\bar{p}_N z - |p_N|^2} \frac{\bar{p}_N - |p_N|^2}{\bar{p}_N - 1}.$$

The proof is completed.   □

4.2. **Unified Results.** In this sub-section, we analyze the energy regulation problem in terms of the delta operator. We factorize the given plant $P(\delta) = [P_1(\delta), P_2(\delta), \ldots, P_n(\delta)]^T$ as

$$P(\delta) = P_s(\delta) H^{-1}(\delta) = \Theta_i(\delta)\Theta_o(\delta) H^{-1}(\delta), \tag{21}$$

where $H(\delta) = B(T\delta + 1)$.

Note that $H(\delta)$ is inner in δ-domain and possesses non-minimum phase zeros $\rho_i \in \bar{\mathbb{D}}_T^c$ at $\rho_i = (p_i - 1)/T$, $i = 1, 2, \ldots, N_p$, in which they also act as the unstable poles of $P(\delta)$.

We consider the following performance index

$$E_\delta^* = \inf_{K \in \mathcal{K}} T \sum_{k=0}^{\infty} \|u(k)\|^2. \tag{22}$$

Note that $\hat{d}_T(\delta) = 1$. Then by invoking the proof of Theorem 2 and (9), we immediately obtain

$$E_\delta^* = \|H^{-1}(\infty)H(\delta) - 1\|_2^2 = \frac{1}{T}\|B^{-1}(\infty)B(z) - 1\|_2^2.$$

We state the analytical expression of E_δ^* for minimum phase case as follows.

Theorem 3. *Suppose that $P(\delta)$ is minimum phase and has unstable poles at $\rho_i \in \bar{\mathbb{D}}_T^c$, $i = 1, 2, \ldots, N_p$. Then*

$$E_\delta^* = \frac{1}{T}\left(\prod_{i=1}^{N_p} |T\rho_i + 1|^2 - 1 \right). \tag{23}$$

To show the convergence, note that the RHS of (23) can be approximated by $2\sum_{i=1}^{N_p} \rho_i$, from Taylor expansion. A fact from spectral mapping theorem says that $\rho_i = (e^{\lambda_i T} - 1)/T$, $i = 1, 2, \ldots, N_p$, with λ_i are the unstable poles of the respecting continuous-time plant $P(s)$. Then we get $\lim_{T \to 0} \rho_i = \lambda_i$. It shows that if T tends to zero then we

completely recover the equivalent continuous-time result (see [3, 5]), i.e.,

$$\lim_{T \to 0} E_\delta^* = 2 \sum_{i=1}^{N_p} \lambda_i = E_c^*,$$

where $E_c^* = \inf_{K \in \mathcal{K}_c} \int_0^\infty \|u(t)\|^2 dt$ is the performance limit for the continuous-time case.

5. Conclusion

We have revisited the $\mathcal{H}_2$ optimal tracking and regulation performance problems for SIMO feedback control systems in terms of delta operator. For the former we modified the existing discrete-time results, and for the latter we first derived a closed form expression for the discrete-time regulation energy performance limits.

We have also shown that the delta-type of the optimal tracking error and regulation energy expressions in δ-domain completely recover the underlying continuous-time results as the sampling period T tends to zero.

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